INVESTIGATION OF THE EFFECT OF A TRANSVERSE ELECTRIC FIELD ON ONE-DIMENSIONAL ELECTROHYDRODYNAMIC FLOW

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The evaluation of the effect of a transverse electric field on the characteristics of an EHD generator is an important problem of electrohydrodynamic energy conversion [1, 2]. This subject was considered in [3, 4] on certain simplifying assumptions, such as: absence of compressibility and zero mobility of charged particles. This paper gives a qualitative analysis of a stationary one-dimensional electrohydrodynamic flow of a compressible gas in the presence of mobile charged particles in a varying electric field having longitudinal and transverse components.

1. Fundamental equations and integrals. Let us consider the stationary motion of a compressible inviscid non-heat-conducting medium with a volume charge in an electric field in a plane channel of constant cross section with walls ($y = \pm h$) of dielectric material. The electrodes are placed at the boot ends of the channel. We shall assume that the channel is narrow, the flow is directed along the x-axis, and the y-axis is normal to the flow. The equations will be averaged with respect to the y-coordinate on the assumption of symmetry of the flow about the channel axis.

Let us estimate the extent of inhomogeneity of the electric field across the channel. We integrate equation $\operatorname{curl} \mathbf{E} = 0$ from $y = 0$ to $y = h$

$$E_x(h) - E_x(0) = \frac{\partial}{\partial x} \int_0^h E_y \, dy$$

Normalizing the fields with respect to the longitudinal field $E_x(0)$ at the channel axis and introducing dimensionless coordinates $x^* = x/L$ and $y^* = y/L$, we obtain

$$\frac{E_x(h) - E_x(0)}{E_x(0)} \sim \frac{\partial E_{yw}}{\partial x^*} \frac{h}{LE_x(0)}$$

Here E_{yw} is the value of transverse field at the channel walls. For a very narrow channel ($h / L \ll 1$) it can be assumed that the variation of the longitudinal field across the channel is small, even when E_{yw} varies along the latter. Generally, the dependence of E_{yw} on x can be determined by solving the problem of a two-dimensioanl flow in the channel. For the qualitative analysis we shall assume in the following that E_{yw} is constant.

The averaging of equation $\operatorname{div} \mathbf{E} = 0$ over the cross section yields

$$E_x' = 4\pi \langle q \rangle - E_{yw} / h \tag{1.1}$$

Here the prime denotes differentiation with respect to x, and $\langle q \rangle$ is the mean density of the electric charge.

The projection of Ohm's law on the x-axis is of the form

$$j_x = q (u + bE_x) \tag{1.2}$$

Equation (1.2) has been written on the assumption that in the lengthwise direction the effect of diffusion is insignificant. The form of Eq. (1.2) is not altered by the averaging, but true values of q and j_x must be replaced in it by their mean values.

The expression for the longitudinal electric force $f_x = qE_x$ will appear in the projection of the equation of hydrodynamic motion on the x-axis. By virtue of the assumption of constancy of the electric field across the channel this expression, after averaging, becomes $\langle f_x \rangle = \langle q \rangle E_x$.

The energy equation contains terms of the form $j_x E_x$ and $j_y E_y$. The mean value of $j_x E_x$ is $\langle j_x \rangle E_x$.

To estimate $j_y E_y$ we examine the projection of Ohm's law on the y-axis with crosswise diffusion taken into account. The term j_y vanishes at the channel axis (owing to the problem symmetry), and at its walls (walls $y = \pm h$ are dielectric). We assume the transverse current j_y to be small in every cross section, i.e. that relationship $qbE_y \sim \sim D\partial q / \partial y$ holds (D is the coefficient of diffusion).

Hence term $j_y E_y$ can be omitted from the energy equation.

In actual electrohydrodynamic installations the transverse current is exactly zero when mobility ($j_y = bqE_y$) is zero. For low mobility the assumption of smallness of j_y may be considered to be in fair agreement with reality. In the case of an electrohydrodynamic accelerator it can be, apparently, assumed that in the presence of a considerable external longitudinal electric field E_0 the motion of charged particles is directed mainly along the channel, hence the assumption of smallness of j_y is valid.

The equation of motion projected on the y-axis is of the form

$$\frac{\partial p}{\partial y} = qE_y$$

The maximum possible value of $f_y = qE_y$ corresponds to those values of the electrical charge density and of field E_y at which a discharge occurs in the gas. In actual installations the relationship

$$\int_0^h f_y \, dy \sim E_n^2 / 8\pi \ll p$$

where E_n is the discharge field, is satisfied even for these values of q and E_y. Hence $\Delta p / p \ll 1$, and the pressure across the channel may be considered as constant.

Equation $\operatorname{div} \mathbf{j} = 0$, after averaging, yields $\langle j_x \rangle = \text{const} = j_0$. In the following the sign of averaging across a cross section will be omitted.

The system of equations for this problem is of the form

$$\rho u = m = \text{const}, \qquad mu' + p' = qE_x, \qquad m\,(c_p T + 0.5\ u^2)' = j_0 E_x \qquad (1.3)$$
$$p = \rho RT, \qquad j_x = q\,(u + bE_x) = j_0, \qquad E_x' = 4\pi q - E_{yw} / h$$

Here ρ is the medium density, u the velocity component along the x-axis ($u > 0$), p the pressure, T the temperature, c_p and R are, respectively, the specific heat at constant pressure and the gas constant, and b is the mobility. To avoid any ambiguity we assume $q > 0$. The system of Eqs. (1.3) is a closed one. Constants m, j_0 and E_{yw} appearing in these equations are considered as given. It is not difficult to derive from Eqs. (1.3) the relationships

$$u' = \frac{qE_x M^2 (u - u_1)}{mu\,(M^2 - 1)}, \qquad M' = \frac{qE_x M^3 (\gamma + 1)(u - u_2)}{2mu^2 (M^2 - 1)}$$

$$u_1 = (\gamma - 1)\, bE_x, \qquad u_2 = (1 + \gamma M^2) / (\gamma + 1), \quad M^2 = \rho u^2 / \gamma p \qquad (1.4)$$

Here M is the gasdynamic Mach number. Using the last of Eqs. (1.3), we eliminate the charge density q from the equations of motion and energy, integrate these, and obtain

$$mu + p - \frac{E_x^2}{8\pi} = \Pi + \frac{E_{yw}}{4\pi h}\int_0^x E_x\, dx$$

$$m(c_pT + 0.5u^2) = \varepsilon + j_0 \int_0^x E_x\, dx$$

Using the equation of state we readily obtain from these relationships the expression for M^2

$$M^2 = \frac{muE_{yw}(\alpha - u)}{4\pi h(\gamma - 1)[j_0(\Pi - mu + E_x^2/8\pi) + (0.5mu^2 - \varepsilon)E_{yw}/4\pi h]}$$

$$\alpha = 4\pi h j_0(\gamma - 1)/\gamma E_{yw} \tag{1.5}$$

2. Determination of flow region boundaries in the uE_x-plane. The last of Eqs. (1.3) together with Eq. (1.5) and the first of Eqs. (1.4) make possible the analysis of motion in the uE_x-plane. To do this we use Eq. (1.5) for tracing curves $M^2 = 0$, $M^2 = 1$ and $M^2 = \infty$ in the uE_x-plane. It is readily seen that the two straight lines $u = 0$ and $u = \alpha$ correspond in the uE_x-plane to $M^2 = 0$.

The Mach number becomes equal to unity or infinity along lines whose equations are of the form

$$au^2 + cE_x^2 + 2du + f = 0 \tag{2.1}$$

When $M^2 = 1$, the coefficients in Eq. (2.1) are

$$a = \frac{m\gamma(\gamma+1)E_{yw}}{8\pi h(\gamma - 1)}, \quad c = \frac{\gamma j_0}{8\pi}, \quad d = -0.5mj_0(\gamma + 1), \quad f = \gamma\left(j_0\Pi - \frac{\varepsilon E_{yw}}{4\pi h}\right)$$

If parameters (j_0, Π, ε, E_{yw}, m, γ, h) of the problem are such that the relationship $\varepsilon E_{yw}/4\pi h > j_0\Pi$ is satisfied, then Eq. (2.1) is that of a real ellipse. If, however, that relationship does not hold, then for Eq. (2.1) to represent a real curve the relation $d^2 > af$ must be satisfied, and, if this relation is not satisfied, a flow at such E_{yw} will not be realized. the center of ellipse $M^2 = 1$ lies at the point defined by the coordinates $E_x = 0$ and $u = \alpha$, and it intersects the axis of abscissas at points $E_x = \pm[8\pi(\varepsilon E_{yw}/4\pi h - j_0\Pi)/j_0]^{1/2}$.

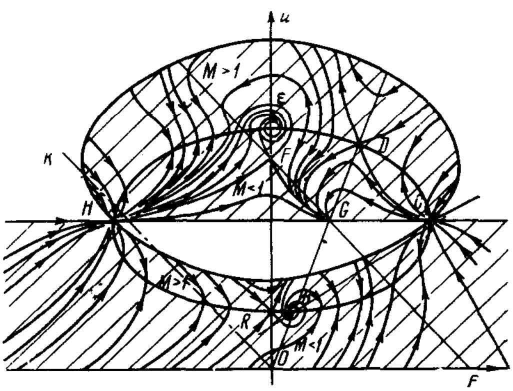

Fig. 1

Depending on the relationship between the problem parameters, the ellipse may have two points of intersection with the axis of abscissas, be tangent to it, or not have any common points with it, i.e. the ellipse may lie above the axis of abscissas. The condition for absence of common points with the axis of abscissas is

$$j_0\Pi > \varepsilon E_{yw}/4\pi h \tag{2.2}$$

When $M^2 = \infty$, the coefficients in Eq. (2.1) are

$$a = mE_{yw} / 8\pi h, \quad c = j_0 / 8\pi, \quad d = -mj_0 / 2, \quad f = j_0\Pi - \varepsilon E_{yw} / 4\pi h$$

The curve $M^2 = \infty$ is an ellipse with its center at point

$$E_x = 0, \quad u = 4\pi h j_0 / E_{yw} = \alpha\gamma / (\gamma - 1).$$

It will be readily seen that the center of ellipse $M^2 = \infty$ lies above that of ellipse $M^2 = 1$. Violation of condition (2.2) indicates the presence of intersection points with the axis of abscissas. The intersection of ellipse $M^2 = \infty$ with line $u = \alpha$, on which lies the center of ellipse $M^2 = 1$, occurs at points coinciding with the extremities of the horizontal semi-axis of the latter.

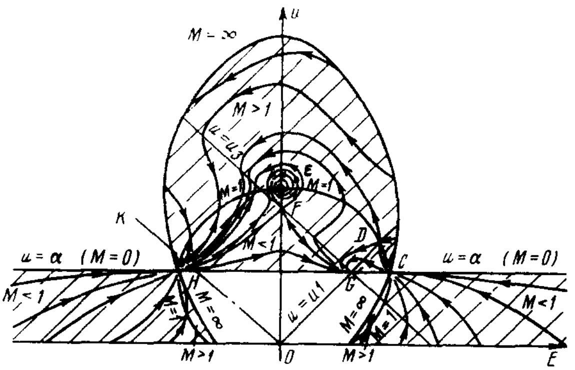

Fig. 2

The possible relative position of lines $M^2 = 0$, $M^2 = 1$ and $M^2 = \infty$ is shown in Figs. 1 and 2. These lines divide the upper half-plane ($u \geqslant 0$) into a number of regions of which those with positive M^2, as defined by Eq. (1.5), have a physical meaning, and are shown hatched in Figs. 1 and 2.

The set of curves $M^2 = \text{const}$ represents a family of ellipses uninterruptedly filling the upper-half plane region between lines $M^2 = 0$ and $M^2 = \infty$, and having common points of intersection with lines $u = 0$ and $u = \alpha$. The Mach number at the intersection points of ellipses is defined by the slope of the integral curve entering that point, i.e. it is equal to the Mach number of an ellipse whose slope at the intersection is equal to the slope of the integral curve.

3. Investigation of singular points. Let us examine the pattern of integral curves of Eqs. (1.3) in the uE_x-plane. The last two of Eqs. (1.3) with the first of Eqs. (1.4) and Eq. (1.5) yield the relationship

$$\frac{du}{dE_x} = \frac{\gamma j_0 E_x (u - u_1)(\alpha - u)}{4\pi(\gamma - 1)(u - u_3)(au^2 + cE_x^2 + 2du + f)} \tag{3.1}$$

Here

$$u_3 = 4\pi h j_0 / E_{yw} - bE_x, \qquad au^2 + cE_x^2 + 2du + f = \gamma(M^2 - 1) / 4\pi h (\gamma - 1)$$

We draw in the uE_x-plane lines $u = u_1 = (\gamma - 1)\, bE_x$ and $u = u_3 = 4\pi h j_0 / E_{yw} - bE_x$. Line $u = u_1$ intersects ellipse $M^2 = 1$ at points B and D; its slope is dependent on the mobility b. When inequality (2.2) is satisfied, i.e. when ellipse $M^2 = 1$ does not intersect the axis of abscissas, there may exist either two intersection points, or tangency, or there may be no points common to ellipse $M^2 = 1$ and line $u = u_1$. If inequality (2.2) is not satisfied, there will always be two intersection points with point D, at which $u > 0$, having a physical meaning. Depending on the slope of line $u = u_1$, point D may lie above, below, or on line $u = \alpha$.

Line $u = u_3$ passes through point F which is the center of ellipse $M^2 = \infty$, and intersects line $u = u_1$ at point G.

Line $u = \alpha$ is horizontal, and on it lies the center of ellipse $M^2 = 1$. It will be readily seen that this line is the integral curve of Eq. (3.1).

Equation (3.1) has eight singular points: A and ε, the intersection points of ellipse $M^2 = 1$ with the coordinate axis $(E_x = 0)$; B and D, the intersection points of ellipse $M^2 = 1$ with line $u = u_1$; F the point of intersection of line $u = u_3$ with the axis of ordinates; G, the intersection point of lines $u = \alpha$ and $u = u_3$; and C and H, the intersection points of ellipse $M^2 = 1$ with line $u = \alpha$. If inequality (2.2) is not satisfied, i.e. when the ellipses intersect the axis of abscissas, points A and B lie in a region devoid of physical meaning. We introduce new variables t and z defined by $u = u^* + z$, $E_x = E_x^* + t$. Here u^* and E_x^* are the coordinates of a singular point.

After linearization in the neighborhood of a singular point Eq. (3.1) becomes

$$z' = \frac{c^\circ t + d^\circ z}{a^\circ t + b^\circ z} \tag{3.2}$$

For point A defined by coordinates

$$E_x(A) = 0, \quad u(A) = \alpha - [\alpha^2 - \gamma(j_0\Pi - \varepsilon E_{yw} / 4\pi h)]^{1/2}$$

the coefficients in Eq. (3.2) will be

$$c^\circ = \frac{j_0 h u(A)}{mE_{yw}(\gamma+1)(4\pi h j_0 / E_{yw} - u(A))} > 0, \quad d^\circ = 0, \quad a^\circ = 0, \quad b^\circ = 1$$

and

$$(a^\circ - d^\circ)^2 + 4b^\circ c^\circ > 0, \qquad a^\circ d^\circ - b^\circ c^\circ < 0$$

In accordance with the generally accepted classification [6] point A is a saddle.

At point ε $\quad E_x(\varepsilon) = 0, \quad u(\varepsilon) = \alpha + [\alpha^2 - \gamma(j_0\Pi - \varepsilon E_{yw} / 4\pi h]^{1/2}$

the coefficients in (3.2) are

$$c^\circ = \frac{j_0 h u(\varepsilon)}{mE_{yw}(\gamma+1)(4\pi h j_0 / E_{yw} - u(\varepsilon))}, \quad d^\circ = 0, \quad a^\circ = 0, \quad b^\circ = 1$$

The coefficient c° is positive, if $4\pi h j_0 / E_{yw} > u(\varepsilon)$, i.e. when point ε lies below point F $(E(F) = 0,\ u(F) = 4\pi h j_0 / E_{yw})$ the center of ellipse $M^2 = 1$. In that case ε is a singular point of the saddle type. If however, $4\pi h j_0 / E_{yw} < u(\varepsilon)$, then $c^\circ < 0$, and point ε is a center.

It follows from the general theory of differential equations that the question whether point ε is a center cannot be solved by using a criterion based on the analysis of only the linear terms in the numerator and the denominator of Eq. (3.1). An examination of the general pattern of integral curves shows that ε is a focal point.

Points F and ε coincide when $4\pi h j_0 / E_{yw} = u(\varepsilon)$, and the velocity at that point is equal u_3. In this case the type of the singular point of Eq. (3.1) cannot be determined by methods of linear theory. We pass in Eq. (3.2) to the new variables z and t, and retain the quadratic terms in the denominator. We have

$$z' = -\frac{\varkappa t}{z(z+bt)}, \quad \varkappa = \gamma u_\varepsilon / 4\pi m(\gamma+1)$$

We look for a solution of this equation in the form of $z = kt$. For the determination of the slope of the characteristic direction in the neighborhood of point ε we obtain the relationship

$$k(k+b) = -\varkappa / z$$

This relationship is valid for $z \to 0$ when $k \to -\infty$, i.e. there exists a characteristic direction with its tangent vertical at point ε.

The coordinates at points D and B are determined by the condition $u = u_1$ and $M^2 = 1$, and are, respectively

$$E(D, B) = \beta \pm [\beta^2 - (j_0\Pi - \varepsilon E_{yw} / 4\pi h) / \delta]^{1/2}, \quad u(D, B) = (\gamma - 1) bE(D, B) \tag{3.3}$$

$$\delta = j_0 / 8\pi + E_{yw} m (\gamma^2 - 1) / 8\pi h, \qquad \beta = m\beta j_0 (\gamma^2 - 1) / \delta$$

The condition of existence of two intersection points in the physical region ($u \geqslant 0$) is

$$\beta^2 > j_0\Pi - \varepsilon E_{yw} / 4\pi h$$

If the ellipses intersect the axis of abscissas, i.e. when inequality (2.2) is not satisfied, only point D lies in the region $u \geqslant 0$. The coordinates in Eq. (3.2) at point B and D are

$$c^\circ = -(\gamma - 1) b, \quad d^\circ = 1, \quad d^\circ = -\gamma, \quad b^\circ = (\alpha - u^*) \omega$$

Here u^* is equal to either $u(D)$ or $u(B)$, depending on the point under consideration

$$\omega = \gamma m (\gamma + 1) E_{yw} / j_0 E^* h (\gamma - 1)$$

Here E^* denotes either $E(B)$ or $E(D)$

$$(a^\circ - d^\circ)^2 + 4b^\circ c^\circ = (\gamma + 1)^2 - 4(\gamma - 1) b (\alpha - u^*) \omega$$

If $u^* \geqslant \alpha$, this expression is positive, and

$$a^\circ d^\circ - b^\circ c^\circ = -\gamma + (\gamma - 1) b\omega (\alpha - u^*) < 0,$$

i.e. the point is of the saddle type. Since inequality $u^* \geqslant \alpha$ can only be satisfied for point D, hence this point, when it lies above or on line $u = \alpha$, is a saddle (Fig. 1).

Depending on the relationship between the parameters, B and D can be either focal, nodal, or saddle points, when they lie below line $u = \alpha$. It can be easily shown that point D, even when it lies below line $u = \alpha$, is always a saddle. Point B can be either a focus, or a node.

Line $u = u_3$ intersects the axis of coordinates ($E_x = 0$) at a point along the coordinate $u(F) = 4\pi h j_0 \quad E_{yw}$. For point F the coefficients in Eq. (3.2) are

$$c^\circ = -j_0^2 h\, u(F) / E_{yw} (\gamma - 1)\, \Omega_1, \; d^\circ = 0, \; a^\circ = b, \; b^\circ = 1$$

Here Ω_1 is the value of $M^2 - 1$ (the left-hand side of Eq. (2.1)) at point F. The sign of coefficient c° depend on whether point F lies inside or outside ellipse $M^2 = 1$. If ellipse $M^2 = 1$ does not intersect the axis of abscissas, i.e. when inequality (2.2) is satisfied, Ω_1 is positive for points lying outside the ellipse, and negative for those inside it. If point F lies inside the ellipse, $(a^\circ - d^\circ)^2 + 4b^\circ c^\circ > 0$, $\quad a^\circ d^\circ - b^\circ c^\circ < 0$, and F is a saddle point (Figs. 1, 2 and 3); if it lies outside ellipse $M^2 = 1$, and if $b^2 > 4\pi h j_0^2\, u(F) / E_{yw} (\gamma - 1)\, \Omega_1$, then point F is a node, while in the case of converse inequality it is a focus.

When point F lies inside ellipse $M^2 = 1$ ($\Omega_1 > 0$) intersecting the axis of abscissas, it is a saddle, and when it is outside such ellipse it is either a focus or a node.

Lines $u = u_1$, $u = u_3$ and $u = \alpha$ intersect a point G ($E_x(G) = \alpha / b(\gamma - 1)$ $u(G) = \alpha$). The type of this point cannot be determined by methods of linear theory. We pass in Eq. (1.3) to variables z and t and retain in the numerator and the denominator the dominant terms

$$z' = -\Omega_3 \frac{z(z-(\gamma-1)bt)}{z+bt} \tag{3.4}$$

Here $\Omega_3 = j_0 E_x(G) h\gamma / (\gamma-1)\Omega_2$, and Ω_2 is the value of M^2-1 (in the left hand side of Eq. (2.1)) for point G. We shall determine the characteristic directions in the neighborhood of point G. To do this we shall look for a solution of Eq. (3.4) in the form of $z = kt$. Substituting into Eq. (3.4), we obtain

$$k = -\Omega_3 \frac{z(k-(\gamma-1)b)}{k+b}$$

For $y \to 0$ we obtain one value $k = 0$ which corresponds to solution $u = \alpha$, and another value, $k \to -b$, i.e. the integral line is tangent to line $u = u_3$.

The coordinates of singular points C and H are, respectively,

$$E(C, H) = \pm\left[\frac{4\pi m(\gamma+1)\alpha}{\gamma} + \frac{8\pi}{j_0}\left(\frac{\varepsilon E_{yw}}{4\pi h} - j_0\Pi\right)\right]^{1/2}$$

The coefficients in Eq. (3.2) are now $c^\circ = 0$, $d^\circ = 1$, $a^\circ = 1$ and $b^\circ = 0$. Points C and H are nodes, since the relationships

$$(a^\circ - d^\circ)^2 + 4b^\circ c^\circ > 0, \qquad a^\circ d^\circ - b^\circ c^\circ > 0$$

are satisfied.

Lines $M^2 = 1$, $E_x = 0$, $u = u_1$, $u = u_3$ and $u = \alpha$ divide the upper half-plane into a number of regions. The sign of du / dE_x, i.e. the sign of the slope of integral curves in each of these can be readily found from Eq. (3.1). Furthermore, the slope of integral curves along the enumerated lines are known. The integral curves intersect lines $E_x = 0$ and $u = u_1$ at zero tangent. Line $u = \alpha$ is the integral curve of Eq. (3.1). Other integral curves can intersect it only at singular points C, G and H. Along lines $u = u_3$ and $M^2 = 1$ the tangents to integral lines are vertical. Having determined the slope of integral curves, the type of their singular points, and the derivatives along singular curves, it becomes possible to obtain in the uE_x-plane a qualitative pattern of behavior of integral curves of the system of Eqs. (1.3). The position of ellipses $M^2 = 1$ and $M^2 = \infty$, and of singular points and lines is given in Figs. 1 and 2.

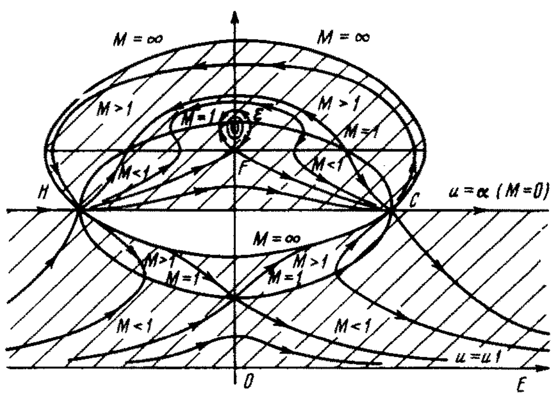

Fig. 3

The case of zero mobility b of charged particles is of particular interest (Fig. 3). The pattern of integral curves represents a particular case of the preceding analysis. Line $u = u_1$ becomes the axis of abscissas, line $u = u_3 = 4\pi h j_0 / E_{yw}$ passing through point F becomes horizontal, and the singular points B, G and D vanish. It immediately follows from Eq. (3.1) that in this case the pattern of integral curves is completely symmetric about the axis of ordinates ($E_x = 0$), hence point ε is necessarily a center.

Using Ohm's law for expressing the electric charge density q and substituting it into the last of Eqs. (1.3), we obtain $$E_x' = \frac{E_{yw}(u_3 - u)}{h(u + bE_x)}$$

This equation together with Eq. (3.1) defines the variation of flow parameters along the channel. The directions of velocity and of the electric field variation are indicated on the integral curves by arrows.

Generally speaking, the motion along the channel (motion in the uE_x-plane as indicated by arrows) can reach the Mach number equal to unity (arrows abutting against ellipse $M^2 = 1$). The situation in which the integral curves do not pass through singular points can only occur at the channel end. A continuous transition through the velocity of sound at points other than singular is not possible. It was shown that points A and D are always singular points of the saddle type, at which transition from the subsonic to the supersonic mode, and vice versa, is generally possible. If point F lies outside ellipse $M^2 = 1$, point ε is also a singular saddle point, and such transitions at it are possible. Points C and H are always nodes. The direction of arrows on integral curves shows that transition through the velocity of sound does not occur there. When B and ε are focal points, the integral curves approaching these will necessarily intersect line $M^2 = 1$ at points other than B and ε, and this corresponds to either the channel end or to a breakdown of the flow continuity. If either inequality (2.2) is not satisfied (Fig. 2), i.e. the ellipses intersect the axis of abscissa, or the mobility b is zero (Fig. 3), the transition through the velocity of sound is only possible at point A. If however the intersection of ellipses with the axis of abscissas occurs in the presence of zero mobility, a transition through the velocity of sound is nowhere possible.

The relationship $u = -bE_x$ is valid along line OK shown in Figs. 1 and 2. To the left of this line the density of the electric current is negative ($j_0 < 0$), while to the right of it, it is positive ($j_0 > 0$). The region comprised between line OK and the axis of ordinates corresponds to a generating process ($j_0E_x < 0$, $j_0 > 0$), while the region of positive E_x, and that to the left of line OK correspond to an acceleration mode ($j_0E_x > 0$). The point at which velocity $u = -bE_x$ is reached must correspond to the channel end, since from the last but one of Eqs. (1.3) follows that at this point $q = \infty$ (u', M' and E' also become infinite).

We note that when E_{yw} tends to zero, ellipses $M^2 = 1$ and $M^2 = \infty$ degenerate into parabolas, and the pattern of integral curves is then of the form given in [5].

The theoretical problem of gas flow under conditions of zero mobility was considered in [4, 7]. The conclusions reached here as regards the solution symmetry about the axis of ordinates ($E_x = 0$), the boundedness of the flow, and the character of the velocity and of the electric field variation along the channel are in agreement with the results cited in [4].

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STABILITY OF COUETTE FLOW IN THE CASE OF A WIDE GAP BETWEEN ROTATING CYLINDERS

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We consider the stability of the Couette flow between two rotating cylinders in the limiting case when the radius of the inner cylinder r_1 tends to zero, and its angular velocity Ω_1 increases to infinity in such a manner that $\Omega_1 r_1^2 = k_1 = \text{const}$.

The dependence of the critical Reynolds number R_* on the wave number α is represented by a neutral curve. The Couette flow loses its stability when the Reynolds number becomes supercritical and $\alpha = 3$. The eigenvector of the linearized problem is computed and used to construct an approximate Taylor vortex.

1. Statement of the problem. A viscous incompressible fluid of unit density and coefficient of viscosity ν fills the space between two concentric cylinders of radii r_1 and r_2 rotating with angular velocities Ω_1 and Ω_2. Letting r_1 tend to zero and Ω_1 to infinity in such a manner that $\Omega_1 r_1^2 = k_1$, we arrive at the limiting flow created by a vortex line of intensity k_1 distributed along the axis of the cylinder whose radius is r_2. Below we study the stablity of this flow.

In Sect. 2 we show that the problem will indeed reach its limiting value when $r_1 \to 0$.

We shall require that there is no loss of fluid across the transverse section. Then the exact solution v_0 of the Navier-Stokes equations satisfying the no-slip conditions at the boundaries represents a Couette flow

$$v_{0r} = v_{0z} = 0, \qquad v_{0\theta} = ar + 1/r, \qquad a = k_2/k_1 - 1, \qquad k_2 = \Omega_2 r_2^2 \qquad (1.1)$$

where r, θ and z denote the cylindrical coordinated.

We shall investigate the stability of the flow (1.1) towards rotationally symmetric perturbations $2\pi / \alpha$-periodic in z. Let us represent the perturbed flow by

$$\mathbf{v}'(r, z, t) = \mathbf{v}_0(r) + e^{\sigma t}\, \mathbf{v}(r, z) \qquad (1.2)$$

Inserting (1.2) into the Navier-Stokes equations and neglecting the quadratic terms,